Matrix Theory: homework #12*
Due 4 December 2013, at start of class

1. True or false? Product of two orthogonal matrices is an orthogonal matrix. Justify.

2. Let V a finite-dimensional real inner product space. Let $\vec{v} \in V$ be a unit vector. A *reflection* with respect to the hyperplane $\{\vec{u} : \vec{u} \perp \vec{v}\}$ is the linear transformation $R_{\vec{v}}\colon V \to V$ defined by

$$R_{\vec{v}}(\vec{w}) = \vec{w} - 2(\vec{w}, \vec{v})\vec{v}.$$

   (a) Prove that every orthogonal operator on V can be written as a product of reflections. (Hint: It might help to draw a picture.)

   (b) Prove that a product of reflections is an orthogonal operator.

3. Let A be a real n-by-n matrix. Show that there is a real number M (depending on A) such that for every natural number r all the entries of A^r are less than M^r in absolute value.

4. Let $T\colon V \to V$ be a linear transformations on a finite-dimensional vector space V. Let W be a T-invariant subspace of V. Prove that the characteristic polynomial of $T|_W$ divides the characteristic polynomial of T.

5. (Bonus problem) Enjoy Thanksgiving.

*This homework is from `http://www.borisbukh.org/MatrixTheory13/hw12.pdf`.