Matrix Theory: homework #1*
Due 4 September 2013, at start of class

Collaboration is permitted, but is advised against, and all collaborators must be acknowledged. Collaboration may involve only discussion; all the writing must be done individually.

The problems appear in no special order. Answers must be fully justified.

1. Prove that $0\vec{v} = \vec{0}$ for any vector space V and any vector $\vec{v} \in V$.

2. Aliens from Sirius have strange rules for manipulating vectors in 2d: the sum of vectors $(x, y)^T$ and $(x', y')^T$ is the vector $(x + x' + 1, y + y' - 1)^T$, whereas the product of a scalar α with a vector $(x, y)^T$ for them is the vector $(\alpha x + \alpha - 1, \alpha y - \alpha + 1)^T$. Is the collection of all 2d vectors a vector space when endowed with these strange operations? If not, which axiom fails?

3. Prove that if two vectors $\vec{v}, \vec{w}$ form a basis for a vector space V, then so do two vectors $\vec{v}, \vec{v} + \vec{w}$.

4. Let V be the collection of all polynomials, and consider the usual operations of addition of two polynomials, and multiplication of a polynomial by a number.
   (a) Explain why V is a vector space.
   (b) Show that there exists no finite set $\{\vec{v}_1, \ldots, \vec{v}_n\}$ that is a basis for V.

5. (Bonus problem) Show that the axioms $1, 2, 5, 6, 7, 8$ in the definition of the vector space in the book do not imply the axioms 3 and 4. To do this, you will need to give an example of a collection of objects and two operations (addition, and scalar multiplication) that satisfy axioms $1, 2, 5, 6, 7, 8$, but fail either 3 or 4 (or both).

*This homework is from `http://www.borisbukh.org/MatrixTheory13/hw1.pdf`.