Math Studies Algebra: homework #6*
Due 12 October 2018, at start of class

Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. For your own learning, you are advised to work individually. Collaboration may involve only discussion; all the writing must be done individually.

Homework must be submitted in LaTeX via e-mail under the same rules as the previous homeworks.

1. Let $a = xyx$, $b = x^2yx^2$, $c = x^3yx^3$ be elements of the free group $F(x,y)$. Prove that the group $\langle a, b, c \rangle \le F(x,y)$ is isomorphic to a free group on three generators.

2. (a) Let X be a finite set, and consider the free group $F(X)$. Show that there are exactly $2^{|X|}$ homomorphisms from $F(X)$ to $\mathbb{Z}/2\mathbb{Z}$.

   (b) Show that the groups $F(x,y)$ and $F(x,y,z)$ are not isomorphic.

3. Show that the group $F(x,y) \times F(x,y)$ is not isomorphic to a free group.