Math Studies Algebra: homework #2*
Due 14 September 2018, at start of class

Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. For your own learning, you are advised to work individually. Collaboration may involve only discussion; all the writing must be done individually.

Homework must be submitted in LaTeX via e-mail. I want both the LaTeX file and the resulting PDF. The files must be of the form `lastname_alg_hwnum.tex` and `lastname_alg_hwnum.pdf` respectively. Pictures do not have to be typeset; a legible photograph of a hand-drawn picture is acceptable.

1. (a) Prove that D_{24} and S_4 are not isomorphic.

   (b) Prove that the additive groups of $\mathbb{Q}$ and $\mathbb{Z}$ are not isomorphic.

2. Let G be a group, and let $\mathrm{Aut}(G)$ be the set of all isomorphisms from G onto G. Recall that $\mathrm{Aut}(G)$ is a group under function composition (called *automorphism group* of G).

   (a) Show that if $G \cong H$, then $\mathrm{Aut}(G) \cong \mathrm{Aut}(H)$.

   (b) (Bonus problem) Does $\mathrm{Aut}(G) \cong \mathrm{Aut}(H)$ imply $G \cong H$?

3. Let ϕ and ψ be two homomorphisms from a group G to another group G'. Let $H = \{x \in G : \phi(x) = \psi(x)\}$. Prove or disprove: H is subgroup of G.

4. Give an example of a group G and of two element $a, b \in G$ such that

   - The order of a and the order of b are finite, and
   - The order of ab is infinite.

5. Prove that if A and B are subsets of G with $A \subseteq B$, then $C_G(B)$ is a subgroup of $C_G(A)$. (Note the reversal in the inclusion direction!).

6. Give an example of a countable group with uncountably many automorphisms. Justify your answer. (Hint: symmetric groups.)

*This homework is from `http://www.borisbukh.org/MathStudiesAlgebra1819/hw2.pdf`.