Math Studies Algebra: homework #23*
Due 20 April 2016, at start of class

Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. For your own learning, you are advised to work individually. Collaboration may involve only discussion; all the writing must be done individually.

Homework must be submitted in LaTeX via e-mail under the same rules as the previous homeworks.

1. (a) Let α be an algebraic integer all of whose conjugates lie on the unit circle $\{z \in \mathbb{C} : |z| = 1\}$. Prove that α is a root of unity. [Consider minimal polynomials of α^n for $n = 1, 2, \dots$]

   (b) (Open problem; extra credit) Let α be an algebraic integer of degree d that is not a root of unity. Prove that there is an algebraic conjugate α' of α satisfying $|\alpha'| \geq 1 + 0.01/d$.

2. Problem 22 in Section 14.6

3. Problem 29 in Section 14.6

4. Problem 4 in Section 14.7