Math Studies Algebra: homework #22*
Due 13 April 2016, at start of class

Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. For your own learning, you are advised to work individually. Collaboration may involve only discussion; all the writing must be done individually.

Homework must be submitted in LaTeX via e-mail under the same rules as the previous homeworks.

1. (a) Let F be a field and let $K = F(x_1, \dots, x_n)$. Construct a subfield L of K such that K/L is Galois with the Galois group S_n.
   (b) Given an arbitrary finite group G construct a Galois extension K/L with Galois group G.
2. Prove that if Galois group of a cubic polynomial f over $\mathbb{Q}$ is cyclic of order 3, then all the roots of f are real.
3. Let F/E and K/F be algebraic field extensions. Show that
$$|\mathrm{Aut}(K/E)| \geq |\mathrm{Aut}(F/E)||\mathrm{Aut}(K/F)|.$$
4. (a) Compute the minimal polynomial of $\sqrt{3} + \sqrt{5}$ over $\mathbb{Q}$.
   (b) Let $f \in \mathbb{Z}[x]$ be the polynomial from part (a). Show that f is reducible in $\mathbb{F}_p[x]$ for every prime p.
5. Find all the Galois conjugates of $\sqrt{2} + \sqrt[3]{3}$ over $\mathbb{Q}$.
6. Exercise 17 in section 14.2
7. Exercise 18 in section 14.2

*This homework is from `http://www.borisbukh.org/MathStudiesAlgebra1516/hw22.pdf`.