Math Studies Algebra: homework #10*
Due 11 November 2015, at start of class

Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. For your own learning, you are advised to work individually. Collaboration may involve only discussion; all the writing must be done individually.

Homework must be submitted in LaTeX via e-mail under the same rules as the previous homeworks.

1. (a) Let K be a field. Let $f \in K[x]$ be a linear polynomial (that is $\deg f = 1$). Show that the quotient ring $K[x]/(f)$ is isomorphic to K. Show also that $K[x]/(f)$ need not isomorphic to K if K is integral domain, but not a field.

   (b) For which quadratic polynomials $f \in \mathbb{R}[x]$ is the ring $\mathbb{R}[x]/(f)$ a field?

2. Problem 26 from Section 7.1 of the textbook (about discrete valuation rings).

3. Problem 27 from Section 7.1 of the textbook (about discrete valuation rings).

4. (a) Show that ideal $(2, x)$ in $\mathbb{Z}[x]$ is not principal.

   (b) (Bonus problem) An ideal is *n-generated* if it is of form (A) for some set A of size n. Let I_n be the ideal $(2^n, 2^{n-1}x, 2^{n-2}x^2, \ldots, x^n)$ in $\mathbb{Z}[x]$. Show that I_n is not n-generated.

5. (Ungraded problem; suggested for the students taking Math Studies Analysis) Let R be the ring of continuous functions from $[0, 1]$ to $\mathbb{R}$ with the usual (pointwise) operations of addition and multiplication. Show that each maximal ideal in R is of the form $\{f : f(x_0) = 0\}$ for some x_0.

*This homework is from `http://www.borisbukh.org/MathStudiesAlgebra1516/hw10.pdf`.