Walk through Combinatorics: homework #4*
Due 5 October 2018, at start of class

Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. For your own learning, you are advised to work individually. Collaboration may involve only discussion; all the writing must be done individually.

Homework must be submitted in LaTeX via e-mail. I want both the LaTeX file and the resulting PDF. The files must be of the form `lastname_discr_hwnum.tex` and `lastname_discr_hwnum.pdf` respectively. Pictures do not have to be typeset; a legible photograph of a hand-drawn picture is acceptable.

1. [1+1] Prove the identity
$$\sum_k k\binom{n}{k} = n2^{n-1}$$
in two ways: by considering an appropriate power series, and by exhibiting a bijection between objects counted by the two sides of the equation.

2. [2] Construct two points sets $P, B \subset \mathbb{R}^2$ such that
   - $|P| = n$ and $|B| = n^{1+o(1)}$,
   - No three points of P lie on a same line,
   - For every two points $a, b \in P$, the (open) line segment (a, b) contains a point of B.

3. [2] For each $m \in \mathbb{N}$ you have m coins worth 2^{m-1} cents. Each of these infinitely many coins is painted in a different color[1]. Let A_n be the number of ways to pay n cents using these coins. For example, $A_4 = 4$ because we can use either both 2-cent coins or one of the three 4-cent coins.

   Let B_n denote the number of ways to write n as a sum of powers of 2. For example, $B_4 = 4$ because $4 = 1 + 1 + 1 + 1$, $4 = 2 + 1 + 1$, $4 = 2 + 2$, $4 = 4$ are all the possible ways of writing 4.

   Prove that $A_n = B_n$.

*This homework is from `http://www.borisbukh.org/DiscreteMath18/hw4.pdf`.

[1]Yes, it is possible. There are continuum many colors.

4. [2+(1 bonus)] Pick any two of the homework problems from homeworks #1 through #3. Generalize the statements and provide solutions. (Bonus will be awarded for particularly nice generalizations.)