Walk through Combinatorics: homework #8*
Due ~~27~~ 29 November 2017, at start of class

Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. For your own learning, you are advised to work individually. Collaboration may involve only discussion; all the writing must be done individually.

Homework must be submitted in LaTeX via e-mail. I want both the LaTeX file and the resulting PDF. The files must be of the form `lastname_discr_hwnum.tex` and `lastname_discr_hwnum.pdf` respectively. Pictures do not have to be typeset; a legible photograph of a hand-drawn picture is acceptable.

1. [2] Let A be a random subset of $[n]$ such that $\Pr[t \in A] = p$ and the events $t \in [n]$ are independent. Let E denote the event that A contains a 4-term arithmetic progression.

   Find a function $t(n)$ such that

   - if $p = o(t(n))$, then $\Pr[E] \to 0$; and
   - if $p = w(t(n))$, then $\Pr[E] \to 1$.

2. [2] Let k be a positive integer. Prove that for every set X of at least $4k^2$ distinct residue classes modulo a prime p, there is an integer a such that the set $\{ax \pmod{p} : x \in X\}$ intersects every interval in $\{0, 1, \ldots, p-1\}$ of length at least p/k.

3. [2] Write down an explicit positive constant c such that the following holds for all natural numbers n. Suppose $a_1, \ldots, a_n$ are n real numbers satisfying $\sum_{i=1}^n a_i^2 = 1$, and $\epsilon_1, \ldots, \epsilon_n \in \{-1, +1\}$ are uniformly and independently chosen signs. Then
$$\Pr\left[\left|\sum_{i=1}^n \epsilon_i a_i\right| \le 1\right] \ge c.$$

4. [Extra credit] Enjoy Thanksgiving!

*This homework is from `http://www.borisbukh.org/DiscreteMath17/hw8.pdf`.