# Walk through Combinatorics: homework #1*
# Due 9 September 2015, at start of class

Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. For your own learning, you are advised to work individually. Collaboration may involve only discussion; all the writing must be done individually.

The number of points for each problem is specified in brackets. The problems appear in no special order.

1. [1] Give an alternative proof of the Erdős–Szekeres theorem using induction.

2. [1+1]

    (a) Let $r$ and $b$ be some fixed natural numbers. Edges of a complete graph on $[n] = \{1, 2, \ldots, n\}$ are colored red and blue. Show that if $n$ is large enough, then the graph contains either an $r$-vertex red clique or a blue path of length $b$ whose vertices are in increasing order.

    (b) Find the smallest $n$ such that (a) holds.

3. [1+1] Let $T$ be a tree having $k$ vertices (a *tree* is a connected graph containing no cycles)

    (a) Prove that if $n > (k - 1)(l - 1)$ and $K_n$ is colored red/blue, then $K_n$ contains a red $T$ or a blue $K_l$.

    (b) Show that this is not true if $n = (k - 1)(l - 1)$.

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