Walk through Combinatorics: homework #6*
Due 4 December 2013

Collaboration and use of external sources are permitted, but discouraged, and must be fully acknowledged and cited. Collaboration may involve only discussion; all the writing must be done individually.

The number of points for each problem is specified in brackets. The problems appear in no special order.

1. [1+1] Let G be a graph, and let H be a graph obtained from G by deleting each edge with probability $1/2$. Let $\chi(G)$ and $\chi(H)$ be the chromatic numbers of G and H.

   (a) Show that $\mathbb{E}[\chi(H)] \geq \chi(G)^{1/2}$. (Hint: consider the complement of H).

   (b) Show that $\Pr[\chi(H) < c\chi(G)^{1/2}] \leq f(c)$ for an explicit function f satisfying $f(c) \to 0$ as $c \to 0$.

2. [2] For a permutation π, let $X = X(\pi)$ be the least number m such that π is a product of m cycles. Show that there is a constant C such that if one picks π uniformly at random from the symmetric group S_n, then

$$\Pr\Big[|X - \mathbb{E}[X]| \geq C\sqrt{n}\Big] \leq 0.01$$

*This homework is from `http://www.borisbukh.org/DiscreteMath13/hw6.pdf`.