Algebraic Structures: homework #13*
Due 4 December 2023, at 9am

Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. You will get most out of the problems if you tackle them on your own. Collaboration may involve only discussion; all the writing must be done individually.

1. Prove that the only subfields of $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ are $\mathbb{Q}, \mathbb{Q}(\sqrt{2}), \mathbb{Q}(\sqrt{3}), \mathbb{Q}(\sqrt{6})$ and $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ itself.

2. Let p and q be distinct prime numbers. Compute the degree of the extension $\mathbb{Q}(\sqrt{p} + \sqrt{q})/\mathbb{Q}$. [Hint: consider also extensions of $\mathbb{Q}$ by $\sqrt{p}$ and $\sqrt{q}$.]

3. Prove that the polynomial $x^3 - 2x + 3$ is irreducible in $\mathbb{Q}(i)[x]$.

*This homework is from `http://www.borisbukh.org/AlgebraicStructuresFall23/hw13.pdf`.