

Algebraic Structures: homework #7*

Due 22 March 2021 at 4:15pm

Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. You will get most out of the problems if you tackle them on your own. Collaboration may involve only discussion; all the writing must be done individually.

1. Let p, q, r be distinct prime numbers. Show that a group G of order pqr contains a normal subgroup of order either p , q , or r .
2. Prove that a group of order 1365 is not simple.
3. Let p be a prime. Show that every group of order p^n contains a subgroup of order p . [Hint: Induction on n . Use the class equation.]
4. Let H be any subgroup of a group G . Define an infinite sequence of subgroups as follows $H_1 = H$, and $H_{i+1} = N_G(H_i)$. Show that if G is finite, then there is an n such that $H_n = H_{n+1} = H_{n+2} = H_{n+3} = \dots$.
5. [Removed]

*This homework is from <http://www.borisbukh.org/AlgebraicStructures21/hw7.pdf>.