Algebraic Structures: homework #6*
Due 15 March 2021 at 4:15pm

Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. You will get most out of the problems if you tackle them on your own. Collaboration may involve only discussion; all the writing must be done individually.

1. Suppose $m \leq n$. Let $G = \{\pi \in S_n : \pi(i) = i \text{ for all } i > m\}$.
   (a) Show that G is a subgroup of S_n isomorphic to S_m.
   (b) Compute the normalizer $N_{S_n}(G)$. [Your answer should be an explicit subgroup of S_n.]
2. Let $m \geq 3$ be an odd integer, and suppose $n \geq m$. Show that every element of A_n is a product of m-cycles. [Hint: examine small m first, and then generalize to all m.]
3. Show that if $n \geq 3$ is odd then the set of all n-cycles in A_n consists of two conjugacy classes of equal size.
4. (a) Let $C_1, \ldots, C_m$ be the conjugacy classes in a finite group G. Show that every automorphism of a group G permutes the set $\{C_1, \ldots, C_m\}$.
   (b) [Removed]
5. Suppose G is a group and $H \leq K \leq G$. Show that $[G : H] = [G : K] \cdot [K : H]$. (Do not assume that G is finite.)

*This homework is from `http://www.borisbukh.org/AlgebraicStructures21/hw6.pdf`.