

Algebraic Structures: homework #10*

Due 12 April 2021 at 4:15pm

Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. You will get most out of the problems if you tackle them on your own. Collaboration may involve only discussion; all the writing must be done individually.

1. Let R be a commutative ring, and I is an ideal in R . Show that the set

$$\{r \in R : \exists n \in \mathbb{Z}_+ r^n \in I\}$$

is an ideal in R .

2. Suppose $\phi: R \rightarrow S$ is a ring homomorphism between a PID R and an arbitrary ring S . Is $\phi(R)$ necessarily PID?
3. Prove that (x, y) is not a principal ideal in $\mathbb{Q}[x, y]$.
4. Prove that the rings $F[x, y]/(y^2 - x)$ and $F[x, y]/(y^2 - x^2)$ are not isomorphic for any field F .
5. Generalize any one problem from homeworks #5 through #9 (except for the problem 5 on homework #5). You must say which problem you are generalizing, state your generalization, and provide a solution to that generalization. [Saying that “ X is a generalization of Y ” means that X implies Y . You do not need to prove that your generalization is indeed a generalization, but it must be a strict generalization, i.e., $X = Y$ is not allowed.]

*This homework is from <http://www.borisbukh.org/AlgebraicStructures21/hw10.pdf>.