

Algebra: homework #8*

Due 31 October 2022

Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. You will get most out of the problems if you tackle them on your own. Collaboration may involve only discussion; all the writing must be done individually.

Homework must be submitted in L^AT_EX via e-mail. I want both the L^AT_EX file and the resulting PDF. The files must be of the form `andrewid_algebra_hwnum.tex` and `andrewid_algebra_hwnum.pdf` respectively. Pictures do not have to be typeset; a legible photograph of a hand-drawn picture is acceptable.

- In the category of groups, give a categorical definition of a kernel of a homomorphism of the form “A kernel of a morphism $\phi: G \rightarrow H$ is a group K and a map $i: K \rightarrow G$ (which we call inclusion map) such that...”. [Since it is a categorical definition, it can refer only to groups and homomorphisms, and not to groups’ elements. It is highly recommend to draw a diagram(s) illustrating the definition.]
 - Reverse the arrows in your definition to get a definition of a “cokernel”. What is it, group-theoretically speaking?
- Suppose $T \in \text{GL}_n(\mathbb{Q})$ satisfies $T^{-1} = T + T^2$. Show that $3 \mid n$.

*This homework is from <http://www.borisbukh.org/Algebra22/hw8.pdf>.