Algebra: homework #4*
Due 26 September 2022

Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. You will get most out of the problems if you tackle them on your own. Collaboration may involve only discussion; all the writing must be done individually.

Homework must be submitted in LaTeX via e-mail. I want both the LaTeX file and the resulting PDF. The files must be of the form `andrewid_algebra_hwnum.tex` and `andrewid_algebra_hwnum.pdf` respectively. Pictures do not have to be typeset; a legible photograph of a hand-drawn picture is acceptable.

1. Recall that we defined group words $w, w' \in W(S)$ to be equivalent if w' can be obtained from w by elementary reductions and expansions. A word $w \in W(S)$ is *reduced* if it does not admit any elementary reductions. For example, xyx^{-1} is reduced, but $xx^{-1}y$ is not.
   (a) Show that $w_1 \sim w_2$ implies that there is a $w \in W(S)$ which reduces to both w_1 and w_2.
   (b) Show that each equivalence class contains a *unique* reduced word. [You may use part (a) even if you do not solve it.]
2. A ring s is *nilpotent of step s* if product of any $s+1$ elements of the ring is equal to 0. Let R be a ring with 1, and let N be a nilpotent subring of R. Show that $G = 1 - N \stackrel{\text{def}}{=} \{1 - n : n \in N\}$ is a nilpotent group under multiplication in R. [Consider the ideal N^i consisting of sums of products of at least i elements of the ring N.]

*This homework is from `http://www.borisbukh.org/Algebra22/hw4.pdf`.