

Algebra: homework #3*

Due 19 September 2022

Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. You will get most out of the problems if you tackle them on your own. Collaboration may involve only discussion; all the writing must be done individually.

Homework must be submitted in L^AT_EX via e-mail. I want both the L^AT_EX file and the resulting PDF. The files must be of the form `andrewid_algebra_hwnum.tex` and `andrewid_algebra_hwnum.pdf` respectively. Pictures do not have to be typeset; a legible photograph of a hand-drawn picture is acceptable.

1. Let p, q, r be distinct prime numbers. Show that a group G of order pqr contains a normal subgroup of order either p , q or r .
2. Let P be a Sylow p -subgroup of a finite group G , and let $N_G(P)$ be its normalizer. Show that $N_G(N_G(P)) = N_G(P)$.
3. Let $G \curvearrowright X$ be an action. Use it to define $G \curvearrowright Y \stackrel{\text{def}}{=} \{(x, x') \in X^2 : x \neq x'\}$ by $g \cdot (x_1, x_2) \stackrel{\text{def}}{=} (g \cdot x_1, g \cdot x_2)$. Suppose that this $G \curvearrowright Y$ is transitive, and that $N \trianglelefteq G$. Show that $N \curvearrowright X$ is either trivial or transitive.

*This homework is from <http://www.borisbukh.org/Algebra22/hw3.pdf>.