

Algebra: homework #2*

Due 12 September 2022

Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. You will get most out of the problems if you tackle them on your own. Collaboration may involve only discussion; all the writing must be done individually.

Homework must be submitted in L^AT_EX via e-mail. I want both the L^AT_EX file and the resulting PDF. The files must be of the form `andrewid_algebra_hwnum.tex` and `andrewid_algebra_hwnum.pdf` respectively. Pictures do not have to be type-set; a legible photograph of a hand-drawn picture is acceptable.

1. Let $(a_1 a_2 \dots a_m)$ be an m -cycle in some symmetric group S_X .
 - (a) Let $\pi \in S_X$ be arbitrary. Show that $\pi(a_1 a_2 \dots a_m)\pi^{-1}$ is also an m -cycle.
 - (b) True or false: if $m \geq 2$, then every permutation in S_n can be written as a product of conjugates of $(a_1 a_2 \dots a_m)$?
2. Let G be a group. Suppose $A \leq G$ is a subgroup of index $[G : A] = n$. Prove that there exists $N \leq A$ such that N is normal in G and $[G : N] \leq n!$. [Consider the action of G on the cosets of A].

*This homework is from <http://www.borisbukh.org/Algebra22/hw2.pdf>.