# Algebra: homework #13*
# Due never

Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. You will get most out of the problems if you tackle them on your own. Collaboration may involve only discussion; all the writing must be done individually.

If you want me to provide feedback, e-mail the homework to me as usual, preferably by Monday, December 12th.

1. (a) Let $F$ be an algebraically closed field. Show that every maximal ideal in $F[x_1, \dots, x_n]$ is of the form $(x_1 - a_1, \dots, x_n - a_n)$ for some $a_1, \dots, a_n \in F$.

   (b) Show that any radical ideal $I \subset F[x_1, \dots, x_n]$ is the intersection of maximal ideals containing it.

2. Let $p$ be a prime number.

   (a) Show that if $f \in \mathbb{Q}[x]$ is irreducible of degree $p$ with $p - 2$ real and 2 non-real zeros, then its Galois group is $S_p$.

   (b) Show that $(x^2 + 4)(x - 2)(x - 4) \cdots (x - 2p + 4) + 2$ is irreducible over $\mathbb{Q}[x]$ and its Galois group is $S_p$.

3. (Challenge problem) Let $F$ be an algebraically closed field of characteristic 0. Let $f \in F[x]$, let $y = f(x)$ and consider the extension $F(x)/F(y)$. Give a necessary and sufficient condition on $f$ for this extension to be cyclic.

*This homework is from `http://www.borisbukh.org/Algebra22/hw13.pdf`.