

# Algebra: homework #12\*

## Due 28 November 2022

Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. You will get most out of the problems if you tackle them on your own. Collaboration may involve only discussion; all the writing must be done individually.

Homework must be submitted in L<sup>A</sup>T<sub>E</sub>X via e-mail. I want both the L<sup>A</sup>T<sub>E</sub>X file and the resulting PDF. The files must be of the form `andrewid_algebra_hwnum.tex` and `andrewid_algebra_hwnum.pdf` respectively. Pictures do not have to be typeset; a legible photograph of a hand-drawn picture is acceptable.

1. Let  $K/F$  be Galois. Suppose that the coefficients of a monic polynomial  $f \in K[x]$  generate  $K$  over  $F$ . Let

$$\mathcal{F} = \{\sigma f : \sigma \in \text{Gal}(K/F)\}.$$

and define polynomial  $g = \prod_{f' \in \mathcal{F}} f'$ .

- Show that  $g \in F[x]$ .
- Show that if  $g$  is irreducible in  $F[x]$ , then  $f$  is irreducible in  $K[x]$ .

2. Happy Thanksgiving!

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\*This homework is from <http://www.borisbukh.org/Algebra22/hw12.pdf>.