Algebra: homework #11*
Due 21 November 2022

Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. You will get most out of the problems if you tackle them on your own. Collaboration may involve only discussion; all the writing must be done individually.

Homework must be submitted in LaTeX via e-mail. I want both the LaTeX file and the resulting PDF. The files must be of the form `andrewid_algebra_hwnum.tex` and `andrewid_algebra_hwnum.pdf` respectively. Pictures do not have to be typeset; a legible photograph of a hand-drawn picture is acceptable.

1. Recall that an extension K/F is separable if every $\alpha \in K$ is a root of a separable polynomial in $F[x]$. Suppose K is a splitting field of a separable polynomial $f \in F[x]$. Show that K/F is separable.

2. Recall the definition of an algebraic integer from homework #10. Let α be an algebraic integer such that $\alpha \in \mathbb{Q}$. Show that $\alpha \in \mathbb{Z}$.

3. Suppose $p \in \mathbb{Z}[x]$ is a monic polynomial, all of whose complex roots are of norm 1. Prove that all the roots of p are roots of unity, i.e., they satisfy $\alpha^r = 1$ for some r. [Hint: let $\alpha_1, \ldots, \alpha_n$ be the roots, and consider the polynomial with roots $\alpha_1^m, \ldots, \alpha_n^m$ for growing $m \in \mathbb{N}$.]

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*This homework is from `http://www.borisbukh.org/Algebra22/hw11.pdf`.