Algebra: homework #7*
Due 25 October 2021, at 10:00am

Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. You will get most out of the problems if you tackle them on your own. Collaboration may involve only discussion; all the writing must be done individually.

Homework must be submitted via Gradescope. It should be typeset, except for the pictures. Pictures and commutative diagrams do not have to be typeset; a legible photograph of a hand-drawn picture is acceptable.

1. Let V be a finite-dimensional vector space. Let σ be the map defined on the elementary tensors in $V^{\otimes n}$ by

$$\sigma(v_1 \otimes v_2 \otimes \cdots \otimes v_{n-1} \otimes v_n) \stackrel{\text{def}}{=} v_n \otimes v_1 \otimes \cdots \otimes v_{n-2} \otimes v_{n-1}.$$

   (a) Show that σ extends uniquely to a linear map $P\colon V^{\otimes n} \to V^{\otimes n}$.

   (b) Let $A_1, \ldots, A_n\colon V \to V$ be linear map on finite-dimensional vector space V. Denoting by tr the trace, show that

$$\operatorname{tr}(A_1 \otimes A_2 \otimes \cdots \otimes A_n \circ P) = \operatorname{tr}(A_n A_{n-1} \cdots A_1).$$

   (c) Prove an analogous formula for $\operatorname{tr}(A_1 \otimes \cdots \otimes A_n)$.

*This homework is from `http://www.borisbukh.org/Algebra21/hw7.pdf`.