Algebra: homework #2*
Due 13 September 2021, at 10:00am

Collaboration and use of external sources are permitted, but must be fully acknowledged and cited. You will get most out of the problems if you tackle them on your own. Collaboration may involve only discussion; all the writing must be done individually.

Homework must be submitted via Gradescope. It should be typeset, except for the pictures. Pictures and commutative diagrams do not have to be typeset; a legible photograph of a hand-drawn picture is acceptable.

1. Let p, q, r be distinct prime numbers. Show that a group G of order pqr contains a normal subgroup of order either p, q or r.

2. Let p be a prime, and let $G = S_X$ be the symmetric group on $X = (\mathbb{Z}/p\mathbb{Z})^2$. Let π be the cyclic permutation defined by $\pi(i, 1) = \pi(i + 1, 1)$ and $\pi(i, j) = (i, j)$ if $j \geq 2$. Let σ be the permutation given by $\sigma(i, j) = (i, j + 1)$.

   (a) Show that that the group generated by π and σ is a Sylow p-subgroup.

   (b) How many Sylow p-subgroups does G have?

*This homework is from `http://www.borisbukh.org/Algebra21/hw2.pdf`.